Edge-Correction for Spatial kernel Smoothing Methods—When Is It Necessary

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Outline

- Spatial kernel smoothing methods;
  - Spatial kernel density estimation;
  - Spatial kernel regression;
- Edge effects & bandwidth
- Comparing edge corrections by simulation;
- Conclusion.

1. Spatial kernel density estimation (KDE)

Based on the spatial data $x_1, x_2, \ldots x_n$ where x_i lie within a two-dimensional region, the KDE of

probability density,

$$\hat{f}(x) = \frac{1}{nh^2} \sum_{i=1}^{n} w\left(\frac{x - x_i}{h}\right),$$

point process intensity,

$$\hat{f}(x) = \frac{1}{h^2} \sum_{i=1}^{n} w\left(\frac{x - x_i}{h}\right).$$

Sometime, $w(x/h)/h^2$ denoted as $w_h(x)$.

2. Spatial kernel regression

Spatial kernel regression assumes existence of a smooth function $r(\cdot)$ relating response y_i to

predictor x_i,

$$y_i = r(x_i) + \varepsilon_i.$$

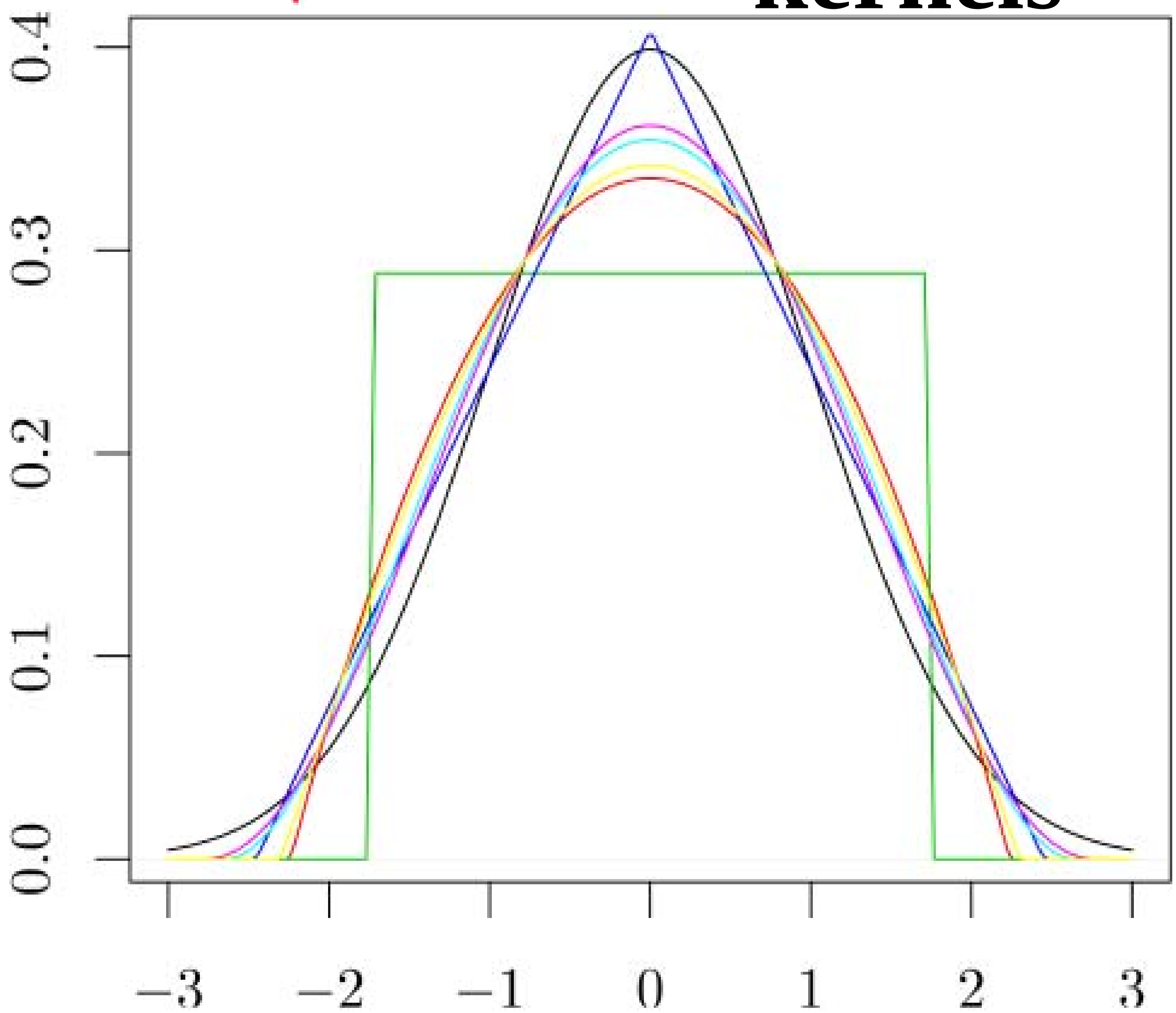
Nadaraya-Watson kernel regression,

$$\widehat{r}(x) = \sum_{i=1}^{n} y_i w' \left(\frac{x - x_i}{h} \right),$$

where

$$w' \left(\frac{x - x_k}{h} \right) = \frac{w\left(\frac{x - x_k}{h}\right)}{\sum_{i=1}^{n} w\left(\frac{x - x_i}{h}\right)}.$$

Diagrams of common kernels



- Gaussian;
- Quartic;
- Rectangular;
- Triangle;
- Biweight;
- Cosine;
- Optcosine.

Edge effects in spatial analysis

- Data observed on region *A*;
- Underline process operates on *B*;
- *A* is a subset of *B*, *A*□ *B*;
- Events on *B\A* may interact with events on *A*.
- KDE without edge-correction will give underestimated results.

Ways to tackle edge effects

- Use of buffer zones;
- Explicit adjustment to take account of unobserved events outside edge;
- When *A* is a rectangle, wrap *A* onto a torus by identifying the opposite edges.

Edge-adjustment approach for KDE

Diggle (1985) and Berman and Diggle (1989) proposed,

$$\widehat{f}(x) = \frac{1}{n}\sum_{i=1}^{n} w_h(x - x_i) \Big/ \int_A w_h(x-u)\,du\ .$$

As for the kernel regression, if we apply the edge adjust factor $1/\int_A w_h(x\text{-}u)\ du$ to both denominator and nominator of the kernel regression estimator, it will be eliminated.

Bandwidth selection in KDE

- Minimising the *mean integrated square error*

$$MISE = E \int |\hat{f}(x;h) - f(x)|^2 \, dx;$$

- Rules of thumb for uni/multivariate KDE (Silverman (1986) and Scott (1992)).

Comparison of KDE with and without edge-correction

- We use the *integrated square error*

$$M = \int_A \left[\hat{f}(x;h) - f(x)\right]^2 dx$$

to measure deviance of the estimated from the true one and

- ratio $R=M_e/M_n$ to measure the effects of doing edge-correction (M_e with edge-correction and M_n without).

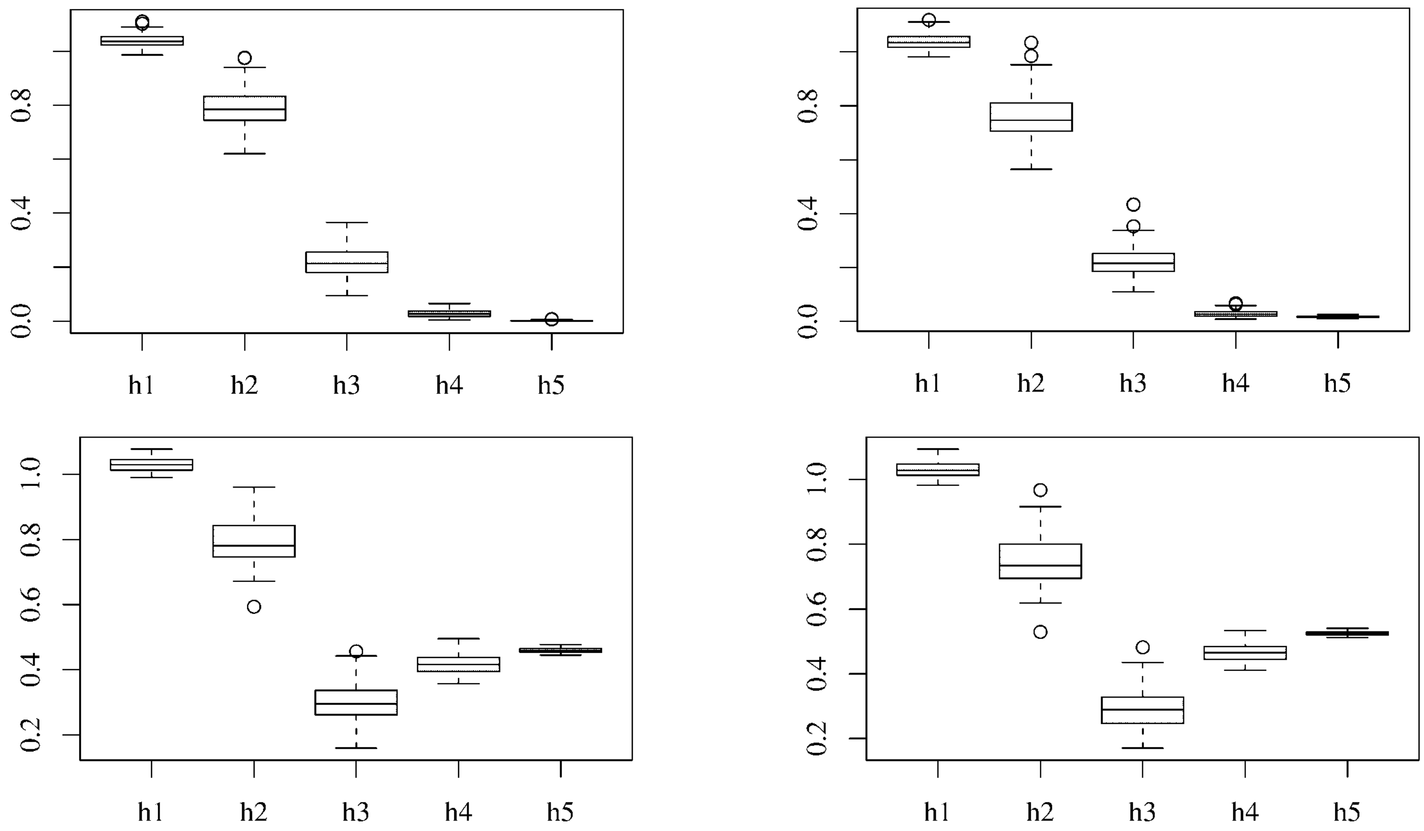
Simulation

- **Model** a homogeneous Poisson process and an inhomonegeous one with intensity proportional to $1+0.7\cos[2\pi(\square\, x\square -0.5)]$, $x\square\, [0,1]\times[0,1]$.
- **Data** simulated 1000 points from the two model;
- **Kernels** Gaussian and quartic;
- **Bandwidths** ¼, ½, 1, 2 and 4 times the value of Scott’s rule of thumb.

Results of *R* in 100 replications

Homogeneous (top) and inhomogeneous (bottom) models

Gaussian (left) and quartic (right) kernels



Conclusion

- For KDE,
  - Selection of kernels is relatively unimportant compared with selection of bandwidth;
  - Could reduce edge problem by reducing bandwidth
  - BUT should not reduce bandwidth because smoothing will not sub-optimal
  - Instead must use correction methods
    - Or collect more data
- For kernel regression the edge effects are much less important than that in KDE.